Locally finding small dominating sets in $K_{2,t}$-minor-free graphs

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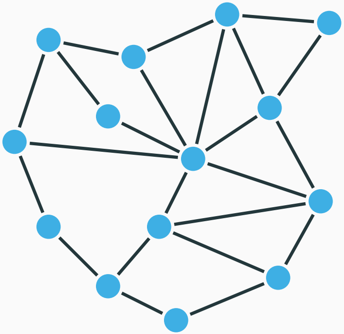
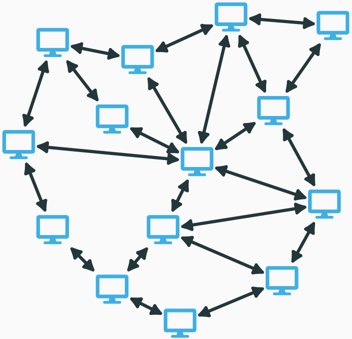
Centralized view

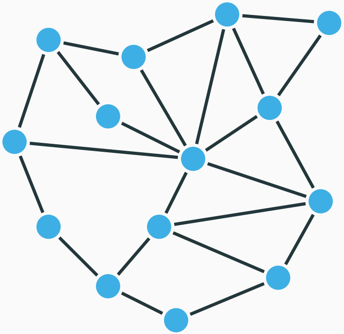


Distributed view



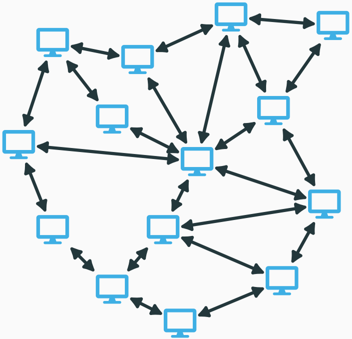
Distributed algorithms

Centralized view



Focused on computing

Distributed view

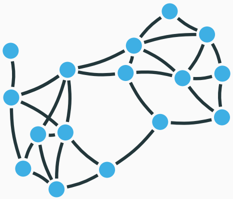


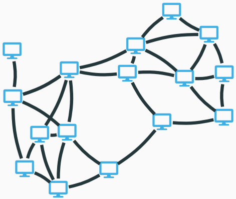
Focused on communication

The LOCAL model

The LOCAL model

graph





network

vertices

+

edges

UID UID

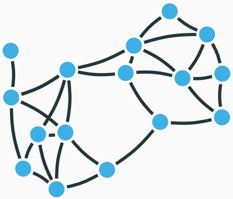
computers

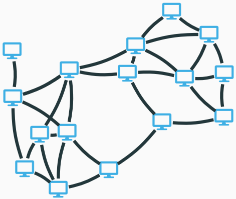
+

connections

The LOCAL model

graph





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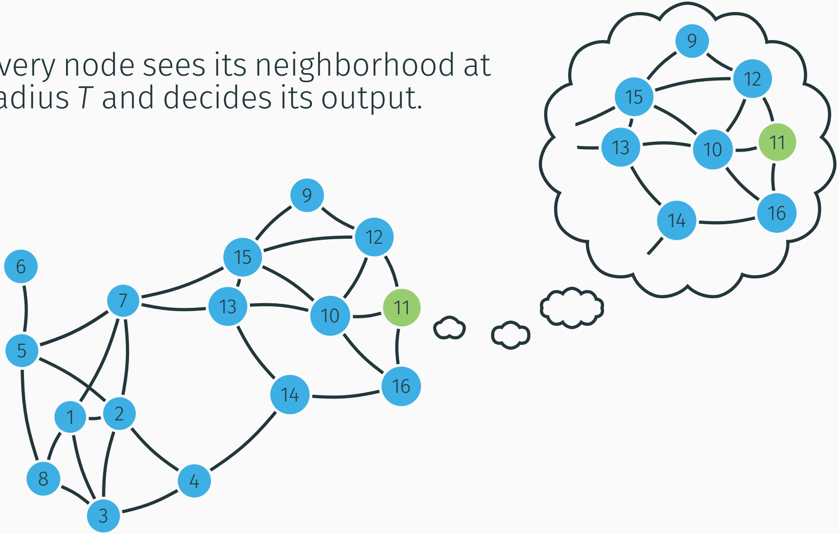
connections

The network is also the input graph!

LOCAL running time T

Every node sees its neighborhood at radius T and decides its output.



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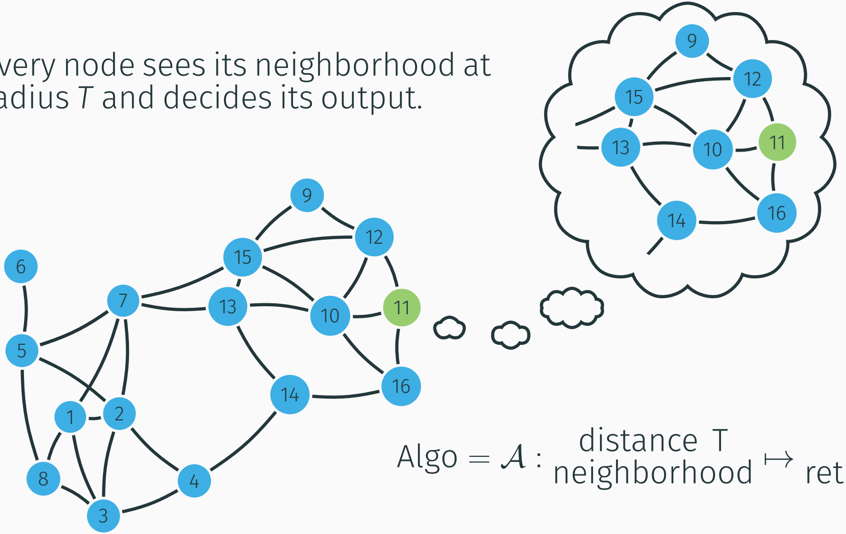
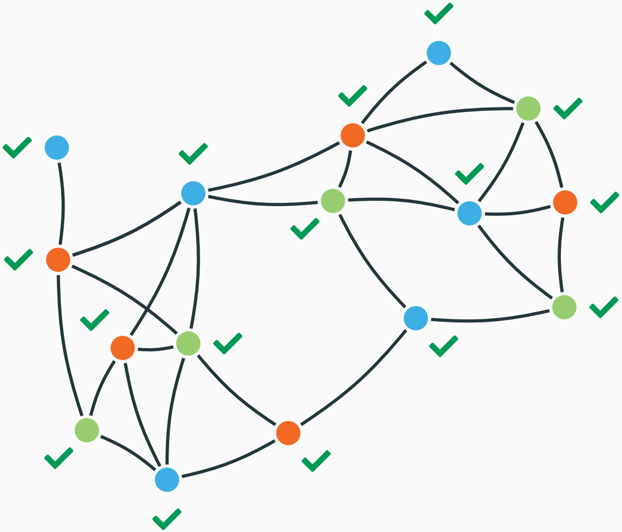


$$\text{Algo} = \mathcal{A} : \begin{array}{c}\text{distance } T\\ \text{neighborhood}\end{array} \mapsto \begin{array}{c}\text{local}\\ \text{return value}\end{array}$$

An example: 3-coloring



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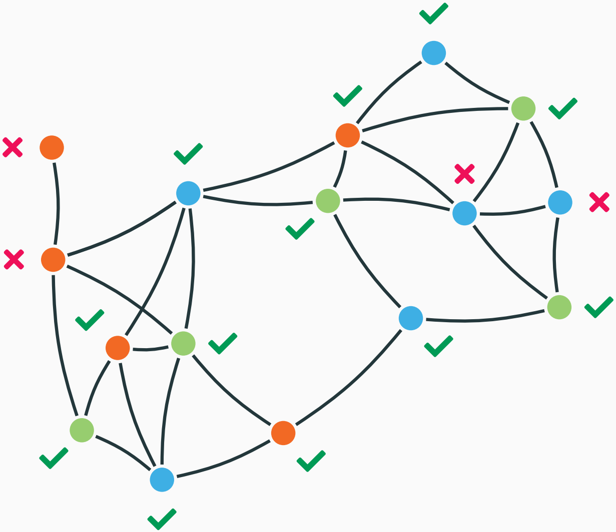
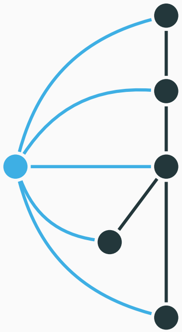


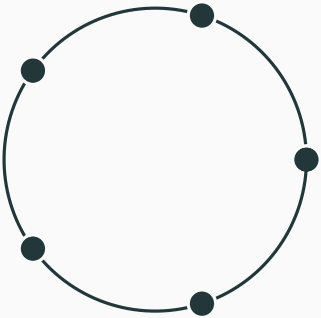
Complexity differences between LOCAL and centralized

Maximum Independent Set
when ∃ universal vertex



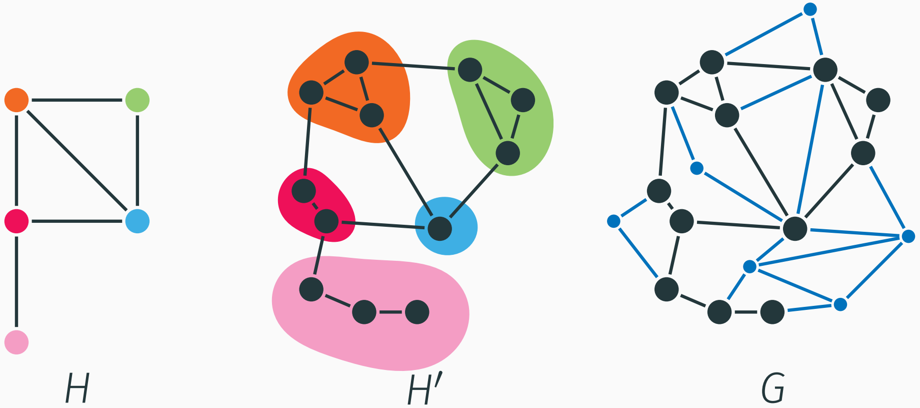
Easy in LOCAL
Hard in centralized

Detecting Cycles



Hard in LOCAL
Easy in centralized

Graph minors



H is a minor of G

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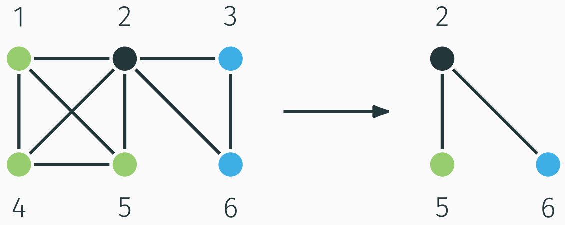
- $K_{2,t}$-minor-free graphs
  - $(2t - 1)$-approximation
  - Generalizes the outerplanar result

The algorithm

The algorithm

- Make G twinless (no vertices s.t. $N[u] = N[v]$)



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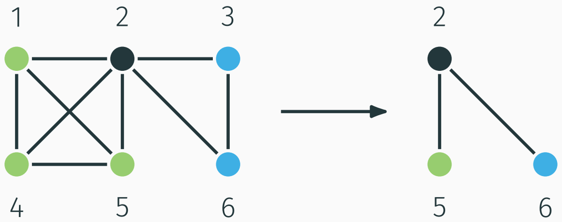
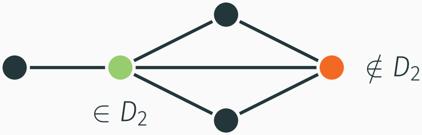


- Return $D_2 = \{v \in V(G) | \nexists u \in V(G - v), N[v] \subseteq N[u]\}$



Approximation factor

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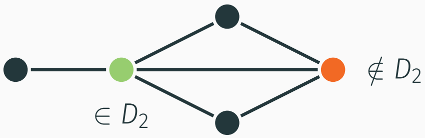


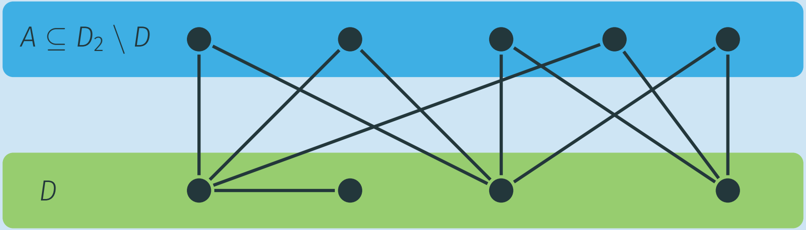
Theorem

Let D a MDS of G. If G is $K_{2,t}$-minor-free, then $|D_2| \leq (2t-1)|D|$.

Part 1: approximation factor

Lemma

Let D a MDS of G. Then $\exists H$ minor of G of the form:

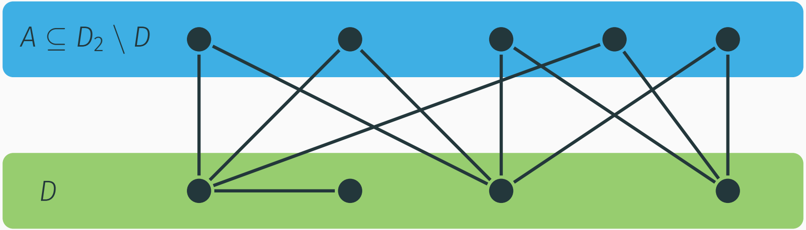


with:

$$|A| \geq \frac{1}{2}|D_2 \setminus D|$$

$$\forall a \in A, |N(a) \cap D| \geq 2$$

Part 2: bound $|D_2 \setminus D|$



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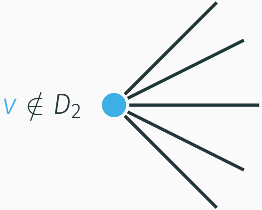
Lemma

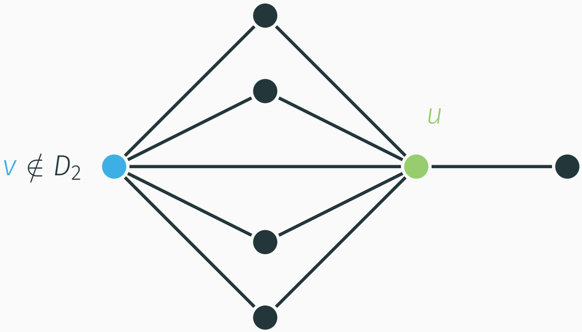
Let H be the previous minor. On a $K_{2,t}$-minor-free graph, $|A| \leq (t-1)|D|$.

Proof 3: D_2 is a dominating set



Proof 3: D_2 is a dominating set

Take u s.t. $N[v] \subsetneq N[u]$ with $N[u]$ maximal.



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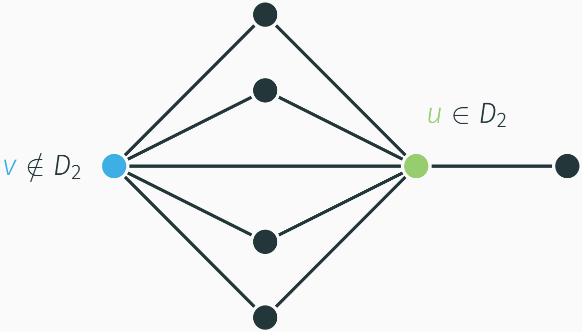

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☻ Thank you! ☻